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# Effects of non-standard couplings, radiative corrections and neutrino masses on the lepton spectra in $\mu$ and $\tau$ decays <sup>1</sup>

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## Abstract

We investigate the combined effect of neutrino masses and non-standard couplings on the various lepton spectra in  $\mu$  and  $\tau$  decays. We emphasize the energy spectra of the neutrinos, which can be measured by secondary reactions in the new KARMEN experiment and hence will yield novel information on deviations from the  $V - A$  structure. To constrain these couplings, QED radiative corrections have to be taken into account. We evaluated them and found small corrections to the neutrino spectra.

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# 1 Introduction

The decay  $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$  is of special interest to both theory and experiment. It is purely leptonic and therefore one can investigate the properties of the weak interactions without QCD complications. In the standard model, the  $V - A$  form of the charged current follows *by construction*. Recently, Fetscher, Gerber and Johnson (FGJ) [1] showed that  $V - A$  in fact emerges with remarkable accuracy from a small set of experiments; nevertheless, some of the non-  $V - A$  couplings could still be substantial. In all of the muon decay experiments up to then, only muon and electron properties had been measured and an ambiguity in nailing down the correct coupling had remained. FGJ had found that it can be resolved by using the cross section of the inverse reaction  $\nu_\mu e^- \rightarrow \mu^- \nu_e$  [1].

A recent experiment [2] allows for the first time to include the information about the  $\nu_e$  from muon decay, in which the neutrinos from  $\mu$  decay undergo a secondary interaction, either  $^{12}C(\nu_e, e^-)^{12}N(g.s.)$  by a charged current process or  $^{12}C(\nu, \nu)^{12}C$  by a neutral current scattering ( $\nu = \nu_e, \bar{\nu}_\mu$ ). This allows to measure the neutrino spectra, both for the electron neutrino (charged and neutral currents) and the muon neutrino (neutral current). As was argued by Fetscher [3, 4], these spectra are sensitive to new interactions which may cause slight deviations from the  $V - A$  behaviour and thus offer an alternative to scattering data. In order to make these meaningful, QED radiative corrections, which are expected to be of the same order of magnitude as possible effects from new physics, must be determined. We therefore compute the radiative corrections to the  $\nu_e$  spectrum.

Also, since  $m_{\nu_\mu} < 270 keV (90\%c.l.)$  [5], its effects could be comparable to those of  $m_e$ . Even larger are the possible effects of  $m_\mu$  and of  $m_{\nu_\tau} < 31 MeV (90\%c.l.)$  [6] in  $\tau$  decays. We have therefore also evaluated the effects of these masses in spectra and rates; we also mention briefly their influence on polarization measurements. Purely leptonic decays with massive neutrinos and arbitrary Lorentz structure were considered by Shrock [7] on a very general basis. Here we focus on those effects of non-zero masses of the  $\nu_\mu$  and  $\nu_\tau$  on spectra and rates which could become measurable in the near future. These effects are due to interference terms and are linear in the neutrino masses. We express the corresponding decay parameters in terms of chiral coupling constants given below, which are best suited to describe the interference.

In order to fix our notation, we write down the general form of the Hamiltonian for the  $\mu$ -decay [1, 8] <sup>2</sup>

$$\mathcal{H} = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T;\varepsilon,\delta=R,L} \{g_{\varepsilon\delta}^\gamma [\bar{e}_\varepsilon \Gamma^\gamma (\nu_e)_n] [(\bar{\nu}_\mu)_m \Gamma_\gamma \mu_\delta] + h.c.\} \quad (1)$$

Here,  $\gamma$  labels the type of interaction (scalar, vector, tensor) and  $\varepsilon$  and  $\delta$  the chirality of the charged leptons. Those of the neutrinos,  $n$  and  $m$  are uniquely fixed by  $\gamma$ ,  $\varepsilon$  and  $\delta$ . The couplings  $g_{\varepsilon\delta}^\gamma$  are normalized as in eq. (2.61) of ref. [8] and their present values or bounds are also given in [8]. In this notation,  $g_{LL}^V = 1$ , all other

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<sup>2</sup> This review contains all necessary background information on  $\mu$  decay

$g_{\varepsilon\delta}^\gamma = 0$ , corresponds to the standard model. The non-standard couplings  $g_{\varepsilon\delta}^\gamma$  arise in extensions of the standard model such as left-right models, extended Higgs structure, dileptons etc. These effects, calculated for  $\mu$  decay, can easily be extended to leptonic  $\tau$ -decays by simply substituting  $\mu \rightarrow \tau$  and  $e \rightarrow e$  or  $\mu$ .

## 2 Mass effects on the decay rate and on the $e^+$ energy spectrum

It is straightforward to calculate the rate for  $\mu$  decay including all mass effects. In the following,  $m_\mu$ ,  $m_e$  and  $m_\nu$  denote the masses of the  $\mu^+$ ,  $e^+$  and  $\bar{\nu}_\mu$ ; the mass of the  $\nu_e$  is always neglected in this paper. Omitting also terms of order  $\alpha \cdot (m_e/m_\mu)$ ,  $\alpha \cdot (m_\nu/m_\mu)$  and  $\alpha \cdot g_{\varepsilon\delta}^\gamma$  (except  $g_{LL}^V$  of course), we obtain

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \left( 1 - 8\varepsilon_e^2 - 8\varepsilon_\nu^2 + 4\eta\varepsilon_e + 4\lambda\varepsilon_\nu + 8\sigma\varepsilon_e\varepsilon_\nu \right) f_W f_r \quad , \quad (2)$$

where  $\varepsilon_e = m_e/m_\mu$ ,  $\varepsilon_\nu = m_\nu/m_\mu$  and  $f_W$  and  $f_r$  are weak and electromagnetic corrections

$$\begin{aligned} f_W &= 1 + \frac{3}{5} \left( \frac{m_\mu}{m_W} \right)^2 \\ f_r &= 1 - \frac{\alpha}{2\pi} \left( \pi^2 - \frac{25}{4} \right) \quad . \end{aligned} \quad (3)$$

In eq. (2), additional terms of order  $O(\varepsilon^3)$  and  $O(\varepsilon^3 \log(\varepsilon))$  have been neglected. The quantities  $\eta$ ,  $\lambda$  and  $\sigma$  are functions of the coupling constants  $g_{\varepsilon\delta}^\gamma$ :

$$\eta = \frac{1}{2} \text{Re} \left\{ g_{LL}^V g_{RR}^{S*} + g_{RR}^V g_{LL}^{S*} + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) + g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) \right\} \quad (4)$$

$$\lambda = \frac{1}{2} \text{Re} \left\{ g_{LL}^S g_{LR}^{S*} + g_{RR}^S g_{RL}^{S*} - 2g_{RR}^V g_{RL}^{V*} - 2g_{LL}^V g_{LR}^{V*} \right\} \quad (5)$$

$$\sigma = \frac{1}{2} \text{Re} \left\{ g_{RR}^S g_{LR}^{V*} + g_{LL}^S g_{RL}^{V*} + g_{LL}^V (g_{RL}^{S*} - 6g_{RL}^{T*}) + g_{RR}^V (g_{LR}^{S*} - 6g_{LR}^{T*}) \right\} \quad . \quad (6)$$

In the standard model  $\eta = \lambda = \sigma = 0$ . With the experimental value in  $g_{LL}^V \approx 1$  we can neglect terms of second order in  $g_{\varepsilon\delta}^\gamma$  ( $g_{\varepsilon\delta}^\gamma \neq g_{LL}^V$ ) and obtain

$$\eta = \frac{1}{2} \text{Re} \left\{ g_{RR}^S \right\} \quad (7)$$

$$\lambda = -\text{Re} \left\{ g_{LR}^V \right\} \quad (8)$$

$$\sigma = \frac{1}{2} \text{Re} \left\{ g_{RL}^S - 6g_{RL}^T \right\} \quad . \quad (9)$$

We note that the present value of  $G_F$  has been derived from the muon decay width  $\Gamma_\mu$  (resp. from the muon lifetime  $\tau_\mu$ ) assuming a pure  $V - A$  interaction.

Without this assumption the experimental error on  $G_F$  increases by a factor of 20 due to the present experimental error on  $\eta$  [8]. The  $\nu_\mu$  interference term  $\lambda\varepsilon_\nu$  can lead to comparable errors on  $G_F$ . Both terms play an even larger role when deriving  $G_F$  from leptonic  $\tau$  decays to test the universality of the charged weak interaction.

Similarly we calculate the  $e^+$  energy spectrum including the effects of the  $e^+$  and  $\bar{\nu}_\mu$  masses. The radiative corrections to this spectrum were explicitly given in ref. [9] for the  $V - A$  case in the relevant limit  $m_e, m_\nu \rightarrow 0$  and we therefore do not discuss them.

Introducing the reduced  $e^+$  energy  $x_e = E_e/W_e$  (with  $W_e \doteq \frac{m_\mu}{2}(1 + \varepsilon_e^2 - \varepsilon_\nu^2)$ ) which varies in the interval

$$x_e^0 \leq x_e \leq 1 \quad , \quad \text{with} \quad x_e^0 = (2\varepsilon_e)/(1 + \varepsilon_e^2 - \varepsilon_\nu^2) \quad , \quad (10)$$

the spectrum  $d\Gamma/dx_e$  reads

$$\frac{d\Gamma}{dx_e} = \frac{G_F^2 W_e^4 m_\mu}{\pi^3} \sqrt{(x_e)^2 - (x_e^0)^2} \left[ H_1 + \frac{2}{9}\rho H_2 + \eta\varepsilon_e H_3 + \lambda\varepsilon_\nu H_4 + \sigma\varepsilon_e \varepsilon_\nu H_5 \right] \quad . \quad (11)$$

With  $t = (1 + \varepsilon_e^2)(1 - x_e) + x_e \varepsilon_\nu^2$  the functions  $H_1, \dots, H_5$  are given by

$$\begin{aligned} H_1 &= \frac{(1 - t + \varepsilon_e^2)(1 - x_e)^2}{t} \\ H_2 &= \left\{ 4t^3 - t^2(5 + 5\varepsilon_e^2 + \varepsilon_\nu^2) + t \left[ (1 - \varepsilon_e^2)^2 - \varepsilon_\nu^2(1 + \varepsilon_e^2) \right] + \right. \\ &\quad \left. + 2\varepsilon_\nu^2(1 - \varepsilon_e^2)^2 \right\} \frac{(1 - x_e)^2}{t^3} \\ H_3 &= \frac{2(1 - x_e)^2}{t} \\ H_4 &= \frac{(1 - t - \varepsilon_e^2)(1 - x_e)^2}{t^2} \\ H_5 &= \frac{(1 + t - \varepsilon_e^2)(1 - x_e)^2}{t^2} \quad . \end{aligned} \quad (12)$$

The parameters  $\eta$ ,  $\lambda$  and  $\sigma$  are listed in eqs. (4), (5) and (6), respectively; the Michel parameter  $\rho$  is

$$\begin{aligned} \rho &= \frac{3}{16} \left\{ |g_{LL}^S|^2 + |g_{LR}^S - 2g_{LR}^T|^2 + |g_{RL}^S - 2g_{RL}^T|^2 + \right. \\ &\quad \left. + |g_{RR}^S|^2 + 4|g_{LL}^V|^2 + 4|g_{RR}^V|^2 \right\} \quad . \end{aligned} \quad (13)$$

### 3 The $\nu_e$ energy spectrum

With the new generation of experiments (KARMEN) [2] the electron neutrino energy spectrum can be measured with a precision of the order of a few percent. This will

yield improved limits to non-standard contributions to muon decay. While the tree level form was given by Fetscher [3, 4], a sensible test of the new interactions requires inclusion of the radiative QED corrections to the standard contribution and of the mass effects of the  $e^+$  and the  $\bar{\nu}_\mu$ .

### 3.1 Radiative QED corrections

The radiative corrections to order  $\alpha$  consist of two parts: Bremsstrahlung processes where a real photon is radiated off the electron or the muon and the virtual corrections (self-energies of the electron and the muon, and the  $\mu - e$  vertex correction) whose interference with the zeroth order process also yields a contribution of order  $\alpha$ . Since the bremsstrahlung photon from the  $\mu$  decay is not detected in the experimental setup, we integrate over all its frequencies.

These corrections have been considered long ago by Behrends, Finkelstein, Kinoshita and Sirlin [9, 10], who applied them to the positron energy distribution. The remarkable fact is, that although the four-Fermi interaction is non-renormalizable, all ultraviolet divergences can be absorbed in the mass of the charged leptons for  $V, A$  interactions.

As we look for a spectrum which is not explicitly given in the literature, we did not make use of the previous work, but performed a new calculation. While taking into account the effects of  $m_e$  and  $m_\nu$  at tree level, we neglect these masses in the radiative corrections.

Both the ultraviolet and the infrared singularities are regularized dimensionally. While the limit  $m_\nu \rightarrow 0$  can be taken at the very beginning of the calculation, we note that individually both, the bremsstrahlung and the virtual corrections are plagued with mass singularities for  $m_e = 0$ . We therefore use a finite electron mass as a regulator. Only when adding virtual and bremsstrahlung corrections the limit  $m_e \rightarrow 0$  is finite due to the KLN theorem [11]. As one of us has discussed the details of the regularization procedure in a similar process at length in ref. [12], we only give the results, separating into bremsstrahlung and virtual corrections to the electron neutrino spectrum.

We define the dimensionless electron neutrino energy  $x$  by

$$E_{\nu_e} = m_\mu \cdot \frac{x}{2} \quad , \quad x \in [0, 1]^3 . \quad (14)$$

The virtual correction is split into an infrared singular and an infrared finite part. We work in  $d = 4 - 2\varepsilon$  dimensions; the singular part is by definition the term proportional to  $1/\varepsilon$ . Writing

$$\frac{d\Gamma^{virt}}{dx} = \frac{d\Gamma_{sing}^{virt}}{dx} + \frac{d\Gamma_{fin}^{virt}}{dx} \quad , \quad (15)$$

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<sup>3</sup>Note, that we neglect  $m_e$  and  $m_\nu$  in radiative corrections in contrast to section 3.2, where tree-level processes are discussed

we have

$$\begin{aligned} \frac{d\Gamma_{sing}^{virt}}{dx} &= -\frac{G_F^2 m_\mu^5 \alpha}{16 \pi^4} \frac{\Gamma(1-\varepsilon) (4\pi)^{3\varepsilon}}{\Gamma^2(2-2\varepsilon)} x(1-x) \times \\ &\times \left[ (1-x) \log(1-x) + 2x - x \log\left(\frac{m_\mu}{m_e}\right) \right] \frac{1}{\varepsilon} \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{d\Gamma_{fin}^{virt}}{dx} &= \frac{G_F^2 m_\mu^5 \alpha}{32 \pi^4} (1-x) \times \\ &\times \left[ 5x(1-x) \log^2(1-x) + (1-6x+9x^2) \log(1-x) + \right. \\ &+ 4x(1-x) \log(1-x) \log(x) + 8x^2 \log(x) + \\ &\left. + 4x(1-x) \text{Li}(x) + x - 13x^2 + A(x) \right] . \end{aligned} \quad (17)$$

Here the Spence function  $\text{Li}(x)$  is defined by

$$\text{Li}(x) = -\int_0^x \frac{dt}{t} \log(1-t) \quad .$$

The term  $A(x)$  in eq. (17) contains the mass singularities ( $m_e \rightarrow 0$ ) and the terms involving the renormalization scale  $\mu$  introduced in dimensional regularization:

$$\begin{aligned} A(x) &= 12x(1-x) \log(1-x) \log\left(\frac{m_\mu}{\mu}\right) + 2x^2 \log^2\left(\frac{m_\mu}{m_e}\right) \\ &- 2x^2 \log(1-x) \log\left(\frac{m_\mu}{m_e}\right) - 12x^2 \log\left(\frac{m_\mu}{\mu}\right) \log\left(\frac{m_\mu}{m_e}\right) \\ &- 4x^2 \log\left(\frac{m_\mu}{m_e}\right) \log(x) + x^2 \log\left(\frac{m_\mu}{m_e}\right) + 24x^2 \log\left(\frac{m_\mu}{\mu}\right) \quad . \end{aligned} \quad (18)$$

Similarly, the bremsstrahlung corrections are split into an infrared singular and an infrared finite part.

$$\frac{d\Gamma^{brems}}{dx} = \frac{d\Gamma_{sing}^{brems}}{dx} + \frac{d\Gamma_{fin}^{brems}}{dx} \quad , \quad (19)$$

with

$$\frac{d\Gamma_{sing}^{brems}}{dx} = -\frac{d\Gamma_{sing}^{virt}}{dx} \quad \text{and} \quad (20)$$

$$\begin{aligned} \frac{d\Gamma_{fin}^{brems}}{dx} &= -\frac{G_F^2 m_\mu^5 \alpha}{192 \pi^4} (1-x) \left[ 6(5x-4x^2) \log^2(1-x) + \right. \\ &+ (11-28x+62x^2) \log(1-x) + 24x(1-x) \log(1-x) \log(x) \\ &+ 48x^2 \log(x) + 12x(2-x) \text{Li}(x) + 24x^2 \text{Li}(1) + \\ &\left. + 11x - \frac{175}{2}x^2 + 6A(x) \right] . \end{aligned} \quad (21)$$

Adding the bremsstrahlung and the virtual corrections we get the total  $O(\alpha)$  correction to the electron neutrino energy spectrum, which in the limit  $m_e \rightarrow 0$  can be written as

$$\frac{d\Gamma^{corr}}{dx} = -\frac{d\Gamma^0}{dx} \frac{\alpha}{2\pi} G(x) \quad , \quad \frac{d\Gamma^0}{dx} = \frac{G_F^2 m_\mu^5 x^2 (1-x)}{16 \pi^3} \quad . \quad (22)$$

Here  $(d\Gamma^0/dx)$  is the standard model spectrum without radiative corrections in the limit  $m_e = m_\nu = 0$ . The function  $G(x)$  reads

$$G(x) = \frac{1}{12} \left[ 12 \log^2(1-x) + \left( \frac{10}{x^2} + \frac{16}{x} + 16 \right) \log(1-x) + 48 \text{Li}(1) + 24 \text{Li}(x) + \frac{10}{x} - 19 \right] \quad . \quad (23)$$

As expected,  $(d\Gamma^{corr}/dx)$  is free from mass singularities. It is very small and in fact the corrections are invisible in a plot of  $(d\Gamma/dx)/\Gamma$ . In contrast, the corrections to the  $e^+$  energy spectrum, given in Fig. 1 of ref. [9], are much bigger. The reason is easy to understand. While  $(d\Gamma^{corr}/dx)$  in eq. (22) is small, the corrections to the  $e^+$  energy spectrum contain rather large logarithms of the form  $\log^2(m_e/m_\mu)$  and  $\log(m_e/m_\mu)$ .

The result (23) is of course also applicable to QCD corrections to the  $e^+$  distribution in semileptonic charm decay in the limit  $m_s \rightarrow 0$ . This has been considered in ref. [13]; however, their function  $G$  does not agree with eq. (23). On the other hand we have checked numerically and analytically that our results agree with those of ref. [14] if  $m_s$  vanishes.

### 3.2 Mass effects on the $\nu_e$ energy spectrum

As the secondary reaction is hardly sensitive to a right-handed  $\nu_e$ , we only calculate the production of a left-handed  $\nu_e$ . Using the scaled energy  $x$  of the  $\nu_e$  defined in eq. (14), we get

$$\frac{d\Gamma_L}{dx} = \frac{G_F^2 m_\mu^5}{16 \pi^3} Q_L^{\nu_e} \{G_1 + \omega_L G_2 + \eta_L \varepsilon_e G_3 + \lambda_L \varepsilon_\nu G_4 + \sigma_L \varepsilon_e \varepsilon_\nu G_5\} \quad . \quad (24)$$

$x$  now varies in the interval  $0 \leq x \leq 1 - (\varepsilon_e + \varepsilon_\nu)^2$ . The functions  $G_1, \dots, G_5$  are most easily expressed in the variables  $r = (1-x)$ ,  $\delta = (\varepsilon_e^2 - \varepsilon_\nu^2)$ ,  $\kappa = (\varepsilon_e^2 + \varepsilon_\nu^2)$  and  $\xi = \sqrt{r^2 - 2\kappa r + \delta^2}$ :

$$\begin{aligned} G_1 &= \frac{(1-r)^2 (r-\kappa) \xi}{r} \\ G_2 &= \frac{2}{9} \frac{(1-r)^2 \xi}{r^3} \left[ -4r^3 + r^2 (1+5\kappa) + r (\kappa - \delta^2) - 2\delta^2 \right] \\ G_3 &= \frac{(1-r)^2 \xi}{r^2} (r-\delta) \end{aligned}$$

$$\begin{aligned}
G_4 &= \frac{(1-r)^2 \xi}{r^2} (r + \delta) \\
G_5 &= \frac{2(1-r)^2 \xi}{r} \quad .
\end{aligned} \tag{25}$$

The parameters  $Q_L^{\nu_e}$ ,  $\omega_L$ ,  $\eta_L$  [4] and the new terms  $\lambda_L$  and  $\sigma_L$  for left-handed  $\nu_e$  read

$$\begin{aligned}
Q_L^{\nu_e} &= \frac{1}{4} \left\{ |g_{RL}^S|^2 + |g_{RR}^S|^2 + 4|g_{LL}^V|^2 + 4|g_{LR}^V|^2 + 12|g_{RL}^T|^2 \right\} \\
\omega_L &= \frac{3}{16} \left\{ |g_{RR}^S|^2 + 4|g_{LR}^V|^2 + |g_{RL}^S + 2g_{RL}^T|^2 \right\} / Q_L^{\nu_e} \\
\eta_L &= \frac{1}{2} \text{Re} \left\{ g_{LL}^V g_{RR}^{S*} + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) \right\} / Q_L^{\nu_e} \\
\lambda_L &= \frac{1}{2} \text{Re} \left\{ g_{RR}^S g_{RL}^{S*} - 2g_{LL}^V g_{LR}^{V*} \right\} / Q_L^{\nu_e} \\
\sigma_L &= \frac{1}{2} \text{Re} \left\{ g_{LR}^V g_{RR}^{S*} + g_{LL}^V (g_{RL}^{S*} - 6g_{RL}^{T*}) \right\} / Q_L^{\nu_e} \quad .
\end{aligned} \tag{26}$$

The radiatively corrected spectrum  $d\Gamma_L^{\text{tot}}/dx$  is now obtained by combining eqs. (24) and (22)

$$\frac{d\Gamma_L^{\text{tot}}}{dx} = \frac{d\Gamma_L}{dx} + \frac{d\Gamma^{\text{corr}}}{dx} \quad . \tag{27}$$

There is an apparent problem in connection with the range of the variable  $x$  in eq. (27). In the radiative corrections  $d\Gamma^{\text{corr}}/dx$ , which have been calculated for  $m_e = m_\nu = 0$ ,  $x$  varies between 0 and 1, whereas in the tree level contribution  $d\Gamma_L/dx$   $x$  varies between 0 and  $(1 - (\varepsilon_e + \varepsilon_\nu)^2)$  due to the finite masses  $m_e$  and  $m_\nu$ . However, to the precision we are working, we are allowed to restrict  $x$  to the common range  $0 \leq x \leq (1 - (\varepsilon_e + \varepsilon_\nu)^2)$  in eq. (27).

In Fig. 1a we compare the radiative corrections (eq. (22)) to the effect of a non-zero  $\omega_L$ , which is obtained from eq. (24) by retaining only the term proportional to  $\omega_L$  in the bracket. For the present experimental upper limit  $\omega_L = 0.1$  [15] (and choosing  $Q_L^{\nu_e} = 1$ ), we see that these are much larger. Only for  $\omega_L \simeq 0.01$  radiative corrections become comparable as illustrated in Fig. 1b.

## 4 The $\bar{\nu}_\mu$ energy spectrum

We also give the energy spectrum of the muon antineutrino which can be measured through neutral current reactions at the secondary vertex. While one expects the  $(V - A)$  term to yield the same tree level spectrum (in the equal mass limit for  $e^+$  and  $\bar{\nu}_\mu$ ) as the one of the positron, the other terms may distort it in different ways. Keeping the  $\bar{\nu}_\mu$ -mass (bounded only by about  $m_e/2$  [5]), one obtains for the energy distribution of the  $\bar{\nu}_\mu$

$$\frac{d\Gamma}{dy} = \frac{G_F^2 m_\mu^5}{16 \pi^3} \left\{ K_1 + \tilde{\omega} K_2 + \eta \varepsilon_e K_3 + \lambda \varepsilon_\nu K_4 + \sigma \varepsilon_e \varepsilon_\nu K_5 \right\} \quad , \tag{28}$$



where the scaled muon antineutrino energy  $y$

$$y = \frac{2E_{\nu_\mu}}{m_\mu(1 + \varepsilon_\nu^2)}$$

varies in the interval

$$\frac{2\varepsilon_\nu}{1 + \varepsilon_\nu^2} \leq y \leq \frac{1 + \varepsilon_\nu^2 - \varepsilon_e^2}{1 + \varepsilon_\nu^2} .$$

The functions  $K_1, \dots, K_5$  are easily written in the variables  $s = (1 - y)(1 + \varepsilon_\nu^2)$ ,  $\zeta = \sqrt{s^2 - 2(1 + \varepsilon_\nu^2)s + (1 - \varepsilon_\nu^2)^2}$  as follows:

$$\begin{aligned} K_1 &= \frac{\zeta}{s} (s - \varepsilon_e^2)^2 (1 - s + \varepsilon_\nu^2) (1 + \varepsilon_\nu^2) \\ K_2 &= \frac{2\zeta}{9s^3} (s - \varepsilon_e^2)^2 (1 + \varepsilon_\nu^2) \times \\ &\quad \times \left[ 4s^3 - s^2 (5 + 5\varepsilon_\nu^2 + \varepsilon_e^2) + s ((1 - \varepsilon_\nu^2)^2 - \varepsilon_e^2(1 + \varepsilon_\nu^2)) + 2\varepsilon_e^2(1 - \varepsilon_\nu^2)^2 \right] \\ K_3 &= \frac{\zeta}{s^2} (1 - s - \varepsilon_\nu^2) (s - \varepsilon_e^2)^2 (1 + \varepsilon_\nu^2) \\ K_4 &= \frac{2\zeta}{s} (s - \varepsilon_e^2)^2 (1 + \varepsilon_\nu^2) \\ K_5 &= \frac{\zeta}{s^2} (1 + s - \varepsilon_\nu^2) (s - \varepsilon_e^2)^2 (1 + \varepsilon_\nu^2) . \end{aligned} \quad (29)$$

The couplings  $\eta$ ,  $\lambda$  and  $\sigma$  are given in eqs. (4), (5) and (6), respectively, while we get for  $\tilde{\omega}$

$$\tilde{\omega} = \frac{3}{4} \left\{ |g_{LL}^V|^2 + |g_{LR}^V|^2 + |g_{RL}^V|^2 + |g_{RR}^V|^2 + 4|g_{LR}^T|^2 + 4|g_{RL}^T|^2 \right\} . \quad (30)$$

Note that  $\tilde{\omega}$  is the analogon of the usual  $\rho$ -parameter.

## 5 Remarks

We have calculated the effects of electromagnetic corrections, masses and non-standard couplings on the rate and spectra in  $\mu$  and  $\tau$  decays. Due to the absence of large logarithms, the radiative corrections to the neutrino energy spectra are very small, unlike those to the spectra of charged leptons. Therefore, the neutrino distributions are in fact suitable for bounding new interactions (see Figs. 1a,b). The new experiments (KARMEN) [2] will measure these spectra in  $\mu$  decay with high precision; preliminary results [15] are indeed very encouraging. Like for the electron, neutrino mass effects are very small in decay rates and spectra (the terms proportional to  $\lambda$  are of relative order  $O(10^{-4})$  in  $\mu$  decay). But as is known [1] for the electron mass term proportional to  $\eta$ , polarization measurements could drastically enhance the sensitivity. For  $\tau$ -decay, however, the mass effects are larger, as  $(\eta\varepsilon_e)$  is

replaced by  $(\eta^\tau m_\mu/m_\tau)$ , where  $\eta^\tau$  is defined for  $\tau$ -decays as in eq. (4). We note that bounds on  $\eta$ ,  $\lambda$  etc. at the level of  $O(\alpha^2)$  are softened by the yet uncomputed electromagnetic two loop corrections to the standard model contribution and the one loop corrections to the non-standard contributions. With the presently attainable experimental precision, however, measurements in muon and leptonic tau decays continue to yield important contributions to our understanding of the fundamental interactions.

## References

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## Figure captions

### Figure 1

Contributions to the  $\nu_e$  spectrum  $d\Gamma/dx$  from muon decay due to non-standard interactions (Fig. 1a:  $\omega_L = 0.1$ , Fig. 1b:  $\omega_L = 0.01$ ) and to radiative corrections as a function of the reduced neutrino energy  $x = E_\nu/m_\mu$ . Radiative corrections have to be taken into account when evaluating experimental spectra at the %-level, but they play a minor role in the region  $x > 0.9$  where the sensitivity for  $\omega_L$  is greatest.

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